Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P61181A

1/1/1/1/1/

P 6 1 1 8 1 A 0 1 2 8

Pearson

Year 2 Work, energy, power - inclined planes, variable resistance, power

1 A van of mass 750 kg is moving up a straight road inclined at an angle β to the horizontal, where $\sin\beta = \frac{1}{21}$. At the instant when the speed of the van is v m s^{-1}, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude λv newtons, where λ is a constant. When the engine of the van is working at a constant rate of 13 kW, the van moves up the road at a constant speed of 20 m s^{-1}.

(*a*) Show that $\lambda = 15$

(4)

Later on, the van is moving along a straight horizontal road. At the instant when the speed of the van is v m s^{-1}, the resistance to the motion of the van is modelled as a force of magnitude $15v$ newtons. When the engine of the van is working at a constant rate of 11.25 kW, the speed of the van is U m s^{-1} and the acceleration of the van is 0.1 m s^{-2}.

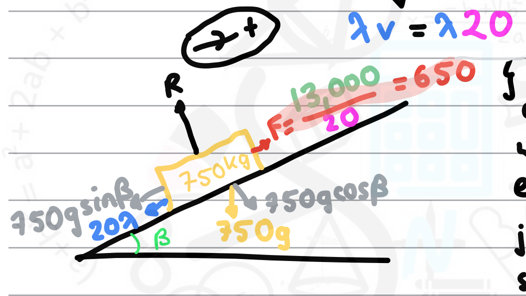
(*b*) Find the value of U.

(4)

(Total for Question 1 is 8 marks)

(a) Let's illustrate the above information on a detailed diagram - label the resistance to motion, direction of motion and the power rearranged

$$\lambda v = \lambda 20$$



{ NOTE: could've done this as a separate line of working but much more efficient in the exam to just add the power rearranged straight onto our diagram

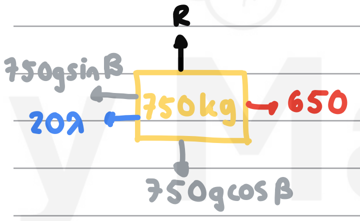
formula: $P = Fv$

$\Rightarrow F = \frac{P}{v}$

where $P = 13\text{kW} \xrightarrow[\text{convert to Watts}]{\times 1000} 13{,}000\text{W}$

and $v = 20\text{ms}^{-1}$

↓ turn the axis



'moving at constant speed' means the van is in equilibrium ∴ forces right = forces left

$$650 = 750g\sin\beta + 20\lambda \leftarrow \text{Solving for this!}$$

and given that $\sin\beta = 1/21$

$$\Rightarrow 650 = \frac{250}{7}g + 20\lambda$$

$$\Rightarrow 20\lambda = 300$$

$\div 20$ $\qquad \div 20$

$$\boxed{\lambda = 15}$$

(b) re-drawing our diagram given the new information:

the resistance to motion, direction of motion and the power

$\underbrace{\text{resistance to motion}}_{15v = 15u}$

$\underbrace{\text{power}}_{\text{rearranged}}$

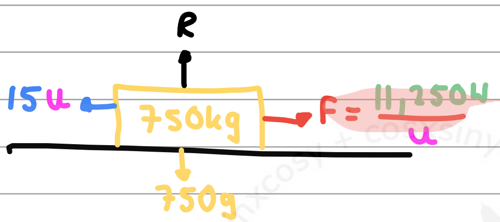
formula: $P = Fv$

$F = \frac{P}{v}$

where

$11.25\text{kW} \xrightarrow[\times 1000]{\text{convert to watts}} 11{,}250\text{W}$

and $v = u$



we are given the acceleration as $a = 0.1\text{ms}^{-2}$ ∴ we can use Newton's Second Law:

formula: $\Sigma F = ma$

Solve for u!

$$\frac{11{,}250}{u} - 15u = 750(0.1)$$

×u ×u

$$15u^2 + 75u - 11{,}250 = 0$$

solve above quadratic for 'u' - using calc equtn solver or quadratic formula:

$$u = \frac{-75 \pm \sqrt{(75)^2 - 4(15)(-11{,}250)}}{2(15)}$$

$$= \frac{-75 \pm \sqrt{680{,}625}}{30}$$

$$= 25 \text{ or } \cancel{-30\text{ms}^{-1}}$$

reject -ve solution

$$\boxed{\therefore u = 25\text{ms}^{-1}}$$

Year 1 Work, energy and power - inclined planes, work-energy principle

2 A small box is projected with speed 7 m s^{-1} from a point O on a fixed rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The box moves up a line of greatest slope of the plane and comes to instantaneous rest at the point A. The coefficient of friction between the box and the plane is $\frac{1}{4}$. In a model of the motion of the box is modelled as a particle.

(*a*) Show that, after coming to rest at A, the box immediately slides back down the plane.

(2)

The speed of the box at the instant when it returns to O is V m s^{-1}.

Given that $OA = \frac{25}{8}$ m,

(*b*) use the work-energy principle to find the value of V.

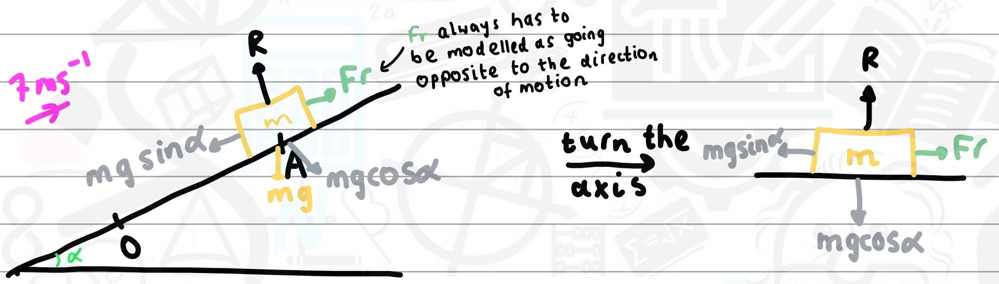
(4)

(*c*) Suggest one way in which the model can be refined to make it more realistic.

(1)

(Total for Question 2 is 7 marks)

(a) let's illustrate the above diagrammatically - label the speed, the friction, the weight



we can tell from the above two diagrams that to show that the box immediately slides down the plane, we need to prove that the left force component exceeds the right i.e

$$mg\sin\alpha > Fr$$

...using our formula for friction to evaluate RHS of above:

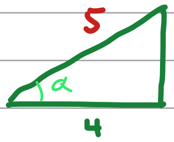
formula: friction $= \mu R$

coeff. of friction (given $\mu = 1/4$)

reaction force

$R(\perp)$: $R = mg\cos\alpha$

given that $\tan\alpha = 3/4$ so constructing the appropriate trig triangle - using the 3,4,5 Pythag. triple :



$\Rightarrow \sin\alpha = 3/5$

$\Rightarrow \cos\alpha = 4/5$

$$\therefore R = mg(4/5)$$
$$= 4/5\, mg$$
$$\Rightarrow Fr = \frac{\cancel{4}}{5} mg\left(\frac{1}{\cancel{4}}\right)$$
$$= \frac{1}{5} mg$$

... evaluating LHS of the inequality:

$$mg\sin\alpha = mg\,(3/5)$$
$$= 3/5\, mg$$

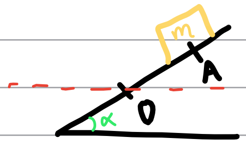
subbing back into our inequality:

$$\frac{3}{5}mg > \frac{1}{5}mg \quad \text{which is true}$$

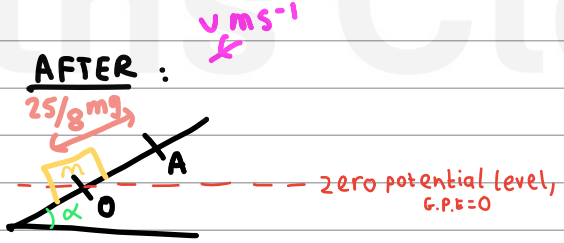
$\boxed{\therefore \text{parcel must slide down the plane}}$

(b) let's draw two diagrams to illustrate this situation - one for BEFORE the parcel travels from A to O and one for AFTER:

BEFORE :

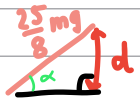


AFTER :



- G.P.E
  ↳ but need perpendicular distance :

i.e



$$\Rightarrow d = \frac{25}{8} mg \sin\alpha$$
$$= \frac{25}{8} mg \left(3/\cancel{5}\right)$$

- K.E
- w.d by friction
  ↳ where we use the fact that $Fr = \frac{1}{5}mg$ from (a)

$$= \frac{15}{8}$$

sub above into the work-energy principle : includes dissipative forces)

$$\underset{\substack{\downarrow \\ \text{n/a}}}{\text{W.d in}} + \underset{\substack{\downarrow \\ \text{kinetic} \\ \text{initial}}}{K.E_i} + \underset{\substack{\downarrow \\ \text{gravitational} \\ \text{potential} \\ \text{initial}}}{G.P.E_i} + \underset{\substack{\downarrow \\ \text{elastic} \\ \text{potential} \\ \text{initial}}}{E.P.E_i} = \underset{\substack{\downarrow \\ \text{kinetic} \\ \text{energy} \\ \text{final}}}{K.E_f} + \underset{\substack{\downarrow \\ \text{gravitational} \\ \text{potential} \\ \text{final}}}{G.P.E_f} + \underset{\substack{\downarrow \\ \text{elastic} \\ \text{potential} \\ \text{final}}}{E.P.E_f} + \text{W.d against friction}$$

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$$

Solve for this !

$$0 + \cancel{m}g\left(\frac{15}{8}\right) + 0 = \frac{1}{2}\cancel{m}(v)^2 + \frac{1}{5}\cancel{m}g\left(\frac{25}{8}\right)$$

cancel m's and simplify

$$\frac{15}{8}g = \frac{1}{2}v^2 + \frac{5}{8}g$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{5}{4}g$$

$$\times 2 \qquad \times 2$$

$$v^2 = \frac{5}{2}g$$

$$\Rightarrow v = \sqrt{5/2\, g} = 4.949...$$

$$\boxed{= 4.9 ms^{-1} (2 d.p)}$$

(c) looking back at Chp 8 Yr 1 Mechanics:

- box is modelled as a particle ∴
  - consider the dimension of the box
  - consider air resistance as box is projected up/sliding down the plane
  - allow for spin

Year 2 Vector Momentum & Impulse - kinetic energy

3 A particle of mass 0.5 kg is moving with velocity $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} when it receives an impulse **I** N s. As a result of the impulse, the kinetic energy of the particle increases by 12 J.

Given that **I** acts in the direction of $(2\mathbf{i} - \mathbf{j})$, find **I**.

(7)

(Total for Question 3 is 7 marks)

---

given the particle's mass, the velocity before and the direction of the impulse, we can use the vector form of the impulse-momentum principle to find the initial velocity, u - let $u = \begin{pmatrix} a \\ b \end{pmatrix}$

formula: $\underline{I} = m(\underline{v} - \underline{u})$

sub into above

$$k\begin{pmatrix} 2 \\ -1 \end{pmatrix} = 0.5\left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}\right)$$

expand above and simplify:

×2 ×2

$$\begin{pmatrix} 4k \\ -2k \end{pmatrix} = \begin{pmatrix} a+1 \\ b-2 \end{pmatrix}$$

...i component: ...b component:

$4k = a+1$ $-2k = b-2$

rearrange for 'a': ...rearrange for 'b':

$\Rightarrow a = 4k-1$ $\Rightarrow b = -2k+2$

$$\therefore v = \begin{pmatrix} 4k-1 \\ -2k+2 \end{pmatrix}$$

now that we've found an expression for 'v', we can apply this to the given information on kinetic energy (i.e that it increases by 12J)

subbing into our formula for change in E_k:

formula: $\frac{1}{2}m(v^2 - u^2)$

first need to work out the scalars of our velocities using Pythagoras':

$|u| = \sqrt{(-1)^2 + (2)^2}$ $|v| = \sqrt{(4k-1)^2 + (2-2k)}^2$

$= \sqrt{5}$ $= \sqrt{(4k-1)^2 + (2-2k)^2}$

now subbing into our ΔE_k formula:

$$\frac{1}{2}(0.5)\left(\left((4k-1)^2+(2-2k)^2\right)^2-(\sqrt{5})^2\right)=12$$

expand and simplify:

what we want to solve for!

$$\frac{1}{4}\left((4k-1)^2+(2-2k)^2-5\right)=12$$

$$\Rightarrow 5k^2-4k-12=0$$

↳ solve for 'k' using calc equtn solver or by factorisation

$ac=-60$

$b=-4$

∴ two numbers that multiply to make -60 and sum to make -4

↳ $-10, 6$

$$\therefore 5k^2-10k+6k-12=0$$

$$5k(k-2)+6(k-2)=0$$

$$\Rightarrow (5k\underset{-ve}{\cancel{+}}6)(k-2)=0$$

$$\therefore k=2$$

now sub this $k=2$ into our expression for $I=k\begin{pmatrix}2\\-1\end{pmatrix}$

$$=2\begin{pmatrix}2\\-1\end{pmatrix}$$

$$\boxed{=\begin{pmatrix}4\\-2\end{pmatrix}}$$

Year 1 Elastic Collisions in 1D - principle of conservation of linear momentum, elastic energy

4 Two smooth spheres, A and B, of the same radius, have masses $2m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal plane. Sphere A is projected towards B with speed u and collides directly with B. The coefficient of restitution between the spheres is e, where $e > \frac{2}{3}$

(*a*) Find, in terms of u and e,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

(*b*) Describe the direction of motion of A immediately after the collision, justifying your answer.

(1)

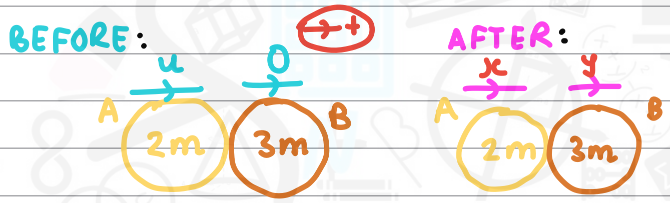
Given that $e = \frac{5}{6}$

(*c*) find the total kinetic energy lost in the collision between A and B.

(4)

(Total for Question 4 is 12 marks)

(a) realising this is an elastic collisions in 1D question - hence illustrating this diagrammatically



and following the usual procedure for these types of collisions:

... first, PCLM i.e momentum before = momentum after

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

subbing into the above:

$$2\cancel{m}(u) + 3\cancel{m}(0) = 2\cancel{m}(x) + 3\cancel{m}(y)$$

expand above and cancel the m's:

$$\Rightarrow 2u = 2x + 3y \quad ①$$

next, NEL:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_B - v_A}{u_A - u_B}$$

subbing into above:

$$e = \frac{y-x}{u}$$

$$\Rightarrow y - x = eu \quad - ②$$

(i) solving ① and ② simultaneously for $v_A = x$ - elim. y

$3\times② - ①$

$$\begin{aligned} -3x + 3y &= 3eu \\ -\quad 2x + 3y &= 2u \\ \hline -5x &= 3eu - 2u \end{aligned}$$

$\div -5 \qquad \div -5$

$$\Rightarrow x = -\frac{u}{5}(3e-2)$$

∴ speed (no direction) of $v_A = x = \boxed{\frac{u}{5}(3e-2)}$

(ii) now solving ① and ② simultaneously for $v_B = y$ (elim. x):

$① - 2\times②$

$$\begin{aligned} 2x + 3y &= 2u \\ + \quad -2x + 2y &= 2eu \\ \hline 5y &= 2u + 2eu \end{aligned}$$

$\div 5 \qquad \div 5$

$$y = \frac{2u}{5}(1+e)$$

$$\therefore v_B = y = \boxed{\frac{2u}{5}(1+e)}$$

(b) we see from part (a) that $v_A < 0 \Rightarrow$ motion of A has been reversed by the collision ($e > 2/3$ so -ve is not cancelled out by another -ve in the brackets) ∴ A travels leftward

(c) we're looking at the kinetic energy lost for both spheres, so will use the formula for $E_k = \frac{1}{2}mv^2$

...first calculating the $K.E_{initial}$:

...for A:	...for B:
$\frac{1}{2}(2m)(u)^2$	$\frac{1}{2}(3m)(0)^2$

$$\therefore K.E_{initial} = mu^2$$

...next, calculating the $K.E_{final}$, where subbing in $e = 5/6$ into v_A and v_B:

$$v_A = -\frac{u}{5}\left(3\left(5/6\right) - 2\right) \qquad v_B = \frac{2u}{5}\left(1 + 5/6\right)$$

$$= -\frac{u}{5}\left(\frac{1}{2}\right) = \frac{-u}{10} \qquad = \frac{11}{15}u$$

... $K.E_{final}$ for A:

$$\frac{1}{2}(2m)\left(-\frac{u}{10}\right)^2$$

$$= \frac{mu^2}{100}$$

... $K.E_{final}$ for B:

$$\frac{1}{2}(3m)\left(\frac{11}{15}u\right)^2$$

$$= \frac{121}{150}mu^2$$

$\therefore$ total $K.E_{final} = \frac{49}{60}mu^2$

$\therefore$ kinetic energy lost $= K.E_{initial} - K.E_{final}$

$$= mu^2 - \frac{49}{60}mu^2$$

$$\boxed{= \frac{11}{60}mu^2}$$

Year 2 Oblique Collisions - spheres

5

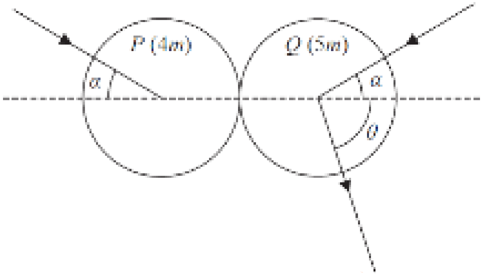


Figure 1

Two smooth uniform spheres, P and Q, with equal radii, are moving on a smooth horizontal plane when they collide. Sphere P has mass $4m$ and sphere Q has mass $5m$. Immediately before they collide, both spheres are moving with the same speed at an angle α, $0° < \alpha < 90°$, to the line joining their centres. Immediately after they collide, Q moves at an angle θ to the line joining their centres, as shown in Figure 1. The coefficient of restitution between the spheres is e.

(*a*) Show that

$$\tan\theta = \frac{9\tan\alpha}{8e - 1}$$

(10)

Given that immediately after the collision, Q moves in a direction that is perpendicular to the line of centres and that $\alpha = 45°$

(*b*) (i) find the value of e,

(ii) find the direction of motion of P immediately after the collision.

(4)

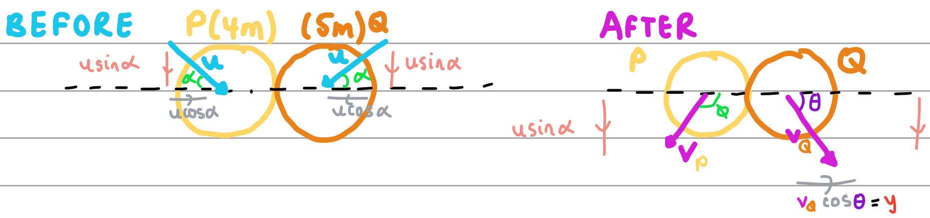
(*c*) Explain how you have used the fact that the two spheres have equal radii in your solution to part (*a*).

(1)

(Total for Question 5 is 15 marks)

to get an expression for $\tan\theta$, we have to find the parallel and perpendicular components of the v_Q

...first let's redraw Fig 1 with the resolved velocities BEFORE and AFTER - let the same speed at which both particles travel initially at $= u$

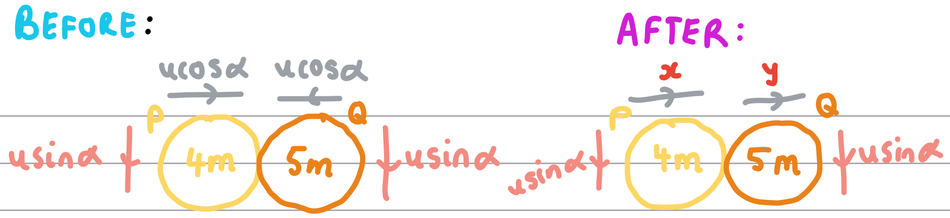


NOTE: impulse does not act perp. to line of centres

∴ no change in the perp. components

$v\sin\theta = u\sin\alpha$

...resolve into i-j components:



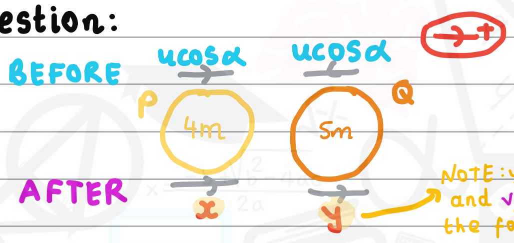
...perpendicular components:

as seen - no Impulse ∴ no change

$v\sin\theta = u\sin\alpha$

... parallel components:

here, the impulse does act along the line of centres - hence treating it as a standard linear collision in 1D question:



NOTE: you could use $v_Q\cos\theta$ and $v_A\cos\theta$ but this makes the following calculations long - it's much easier to work with x and y for the velocities after

(and you don't need to deal with an additional angle)

...first, applying the above to PCLM - i.e that momentum before = momentum after:

formula: $m_Pu_P + m_Qu_Q = m_Pv_P + m_Qv_Q$

subbing into the above:

$$4\cancel{m}(u\cos\alpha) + 5\cancel{m}(-u\cos\alpha) = 4\cancel{m}x + 5\cancel{m}y$$

cancel the m's and expand the brackets:

$$4x + 5y = -u\cos\alpha \quad -①$$

...next, applying this to NEL:

formula: $e = \dfrac{\text{speed of separation}}{\text{speed of approach}} = \dfrac{v_Q - v_P}{u_Q - u_P}$

Sub into above:

$$e = \frac{y - x}{u\cos\alpha - (-u\cos\alpha)} = \frac{y - x}{2u\cos\alpha}$$

$$\Rightarrow y - x = 2eu\cos\alpha \quad ②$$

solve ① and ② simultaneously for y (elim. x)

② ×4 + ①

$$\begin{aligned} -4x + 4y &= 8eu\cos\alpha \\ +\;4x + 5y &= -u\cos\alpha \\ \hline 9y &= 8eu\cos\alpha - u\cos\alpha \end{aligned}$$

factorising $u\cos\alpha$ out and ÷9

÷9 ÷9

$$\Rightarrow y = \frac{u\cos\alpha}{9}(8e-1)$$

∴ looking at our very first 'after' diagram:

$$\tan\theta = \frac{\text{perpendicular}}{\text{parallel}}$$

$$\tan\theta = \frac{u\sin\alpha}{\frac{u\cos\alpha}{9}(8e-1)}$$

×9 ×9

$$\Rightarrow \tan\theta = \frac{9u\sin\alpha}{u\cos\alpha(8e-1)}$$

and using the fact that $\frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

$$\boxed{\Rightarrow \tan\theta = \frac{9\tan\alpha}{8e-1}}$$

(b)(i) the fact that the particle Q moves in a direction that is perpendicular to the line of centres implies that the parallel comp. of $V_Q = 0$

$$\Rightarrow \frac{u\cos\alpha}{9}(8e-1) = 0$$

$$\Rightarrow 8e - 1 = 0$$

$$\Rightarrow 8e = 1$$

÷8 ÷8

$$\Rightarrow \boxed{e = 1/8}$$

(ii) for the direction of motion of Q, we need to find the angle 'ø':

... perp.component :

·no Impulse ∴ no change

$\Rightarrow v_p \sin\phi = u\sin\alpha$

... parallel component:

↳ treat as a standard direct collision in 1D question

∴ solve the ① and ② from (a) simultaneously - this time for 'x' (elim. 'y'):

②×5−①

$$-5x+5y=10eu\cos\alpha$$
$$-\quad 4x+5y=-u\cos\alpha$$
$$-9x \quad =10eu\cos\alpha+u\cos\alpha$$

factorise ucosα out and ÷−9 ÷−9

÷−9

$$\Rightarrow x = -\frac{u}{9}\cos\alpha(10e+1)$$

but given that $e=1/8$, sub into above:

$$x = -\frac{u}{9}\cos\alpha(10(1/8)+1)$$
$$= -\frac{1}{4}u\cos\alpha$$

∴ looking at our first 'after' diagram:

$$\phi = \tan^{-1}\left(\frac{u\sin\alpha}{\frac{1}{4}u\cos\alpha}\right)$$
$$=\tan^{-1}(4\tan\alpha)$$
$$=\tan^{-1}(4\tan(45°))$$
$$=\tan^{-1}(4)$$
$$=76°$$

$\boxed{\therefore P \text{ travels } 76° \text{ to the line of centres}}$

(c) impulse between spheres acts horizontally i.e along line of centres

Year 2 Elastic Strings and Springs - equilibrium and dynamic problem, work-energy principle

6 Two fixed points, A and B, lie on a horizontal ceiling with $AB = 6a$. A light elastic string of modulus of elasticity $\frac{5mg}{3}$ has one end attached to A and the other end attached to B.

A particle P of mass $4m$ is attached to the midpoint of the string and P hangs in equilibrium at a distance $4a$ below AB.

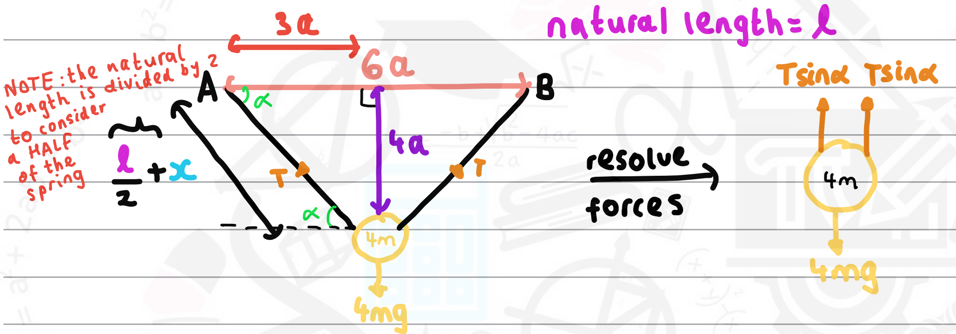
(*a*) Show that the natural length of the string is $4a$. **(5)**

The particle P is now held at the midpoint of AB and released from rest.

(*b*) Find the maximum speed of P as it falls. **(6)**

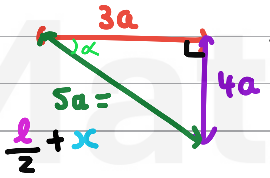
(Total for Question 6 is 11 marks)

(a) notice this is an EQUILIBRIUM PROBLEM involving strings and springs hence the most important thing is to draw a detailed diagram; let natural length $= l$



looking at the above string diagram, we can exploit triangle properties and find the value of the hypothenuse using PYTHAGORAS':

...from above (we can exploit 3,4,5 Pythag. triple):



from this, we can derive:

$$\sin\alpha = \frac{O}{H} = \frac{4\cancel{a}}{5\cancel{a}} = \frac{4}{5}$$

$$\cos\alpha = \frac{A}{H} = \frac{3\cancel{a}}{5\cancel{a}} = \frac{3}{5}$$

$$\therefore \frac{l}{2}+x = 5a$$

also we can exploit equilibrium to get the T needed to eventually get our l

↳ consider the force diagram:

$R(\uparrow)$: $2T\sin\alpha = 4mg$

and know that $\sin\alpha = 4/5$ (from trig triangle) - subbing this in:

$$2T\left(4/5\right) = 4mg$$

$\div 8/5$ $\quad\quad$ $\div 8/5$

$$\Rightarrow T = \frac{5}{\cancel{8}_{2}}(\cancel{4}mg)$$

$$\Rightarrow T = \frac{5}{2}mg$$

and subbing this into our formula for elastic strings and springs (Hooke's law)

mod. of elasticity ↘ ↙ extension

formula: $T = \frac{\lambda x}{l}$ ← natural length

subbing into the above (but the info on HALF a string!)

$$\frac{5}{2}\cancel{mg} = \frac{\frac{5\cancel{mg}}{3}\left(5a - \frac{l}{2}\right)}{\frac{1}{2}l}$$

cancel mg's

$$\frac{5}{2} = \frac{\frac{5}{3}\left(5a - \frac{l}{2}\right)}{\frac{1}{2}l}$$

$\times\frac{1}{2}l$ $\quad\quad$ $\times 1/2\, l$

$$\frac{5}{4}l = \frac{5}{3}\left(5a - \frac{l}{2}\right)$$

expand and solve for 'l':

$$\frac{25}{3}a = \frac{5}{4}l + \frac{5l}{6}$$

$$\Rightarrow \frac{25}{12}l = \frac{25}{3}a$$

$$\Rightarrow \boxed{l = 4a}$$

(b) now we have a DYNAMICS problem - want to find the max. speed of the string - this only occurs when object is in equilibrium (part (a)) - drawing a detailed BEFORE and AFTER diagram (and label with appropriate energies:

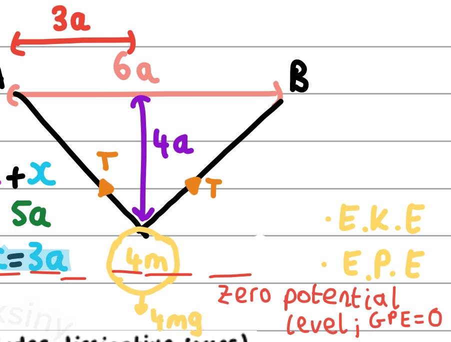
BEFORE:

$\therefore x = 3a - \frac{4l}{2}$

$= a$

- E.P.E
- G.P.E

AFTER:



$2a + x$

$= 5a$

$\Rightarrow x = 3a$

- E.K.E
- E.P.E

sub above into the work-energy principle : includes dissipative forces)

w.d in + K.E$_i$ + G.P.E$_i$ + E.P.E$_i$ = K.E$_f$ + G.P.E$_f$ + E.P.E$_f$ + w.d against friction

w.d in → n/a; K.E$_i$ → kinetic initial; G.P.E$_i$ → gravitational potential initial; E.P.E$_i$ → elastic potential initial; K.E$_f$ → kinetic energy final; G.P.E$_f$ → gravitational potential final; E.P.E$_f$ → elastic potential final

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$$

multiply by 2 as evaluate above for the whole spring!

$$0 + 4\cancel{m}g(4a) + \frac{\frac{5}{3}\cancel{m}g(2a)^2}{2(4a)} = \frac{1}{2}(4\cancel{m})(v^2) + \frac{\frac{5}{3}mg(2\times3a)^2}{2(4a)}$$

cancel m's:

$$16ag - \frac{1}{2}(4)(v^2) = \frac{5/3\,g}{8a}\left((6a)^2 - (2a)^2\right)$$

simplify to solve for v^2:

$$16ag - 2v^2 = \frac{5}{24a}(32a^2)$$

$$\Rightarrow 2v^2 = 16ag - \frac{20ag}{3}$$

$$\Rightarrow 2v^2 = \frac{28}{3}ag$$

$\div 2$ $\quad$ $\div 2$

$$v^2 = \frac{14}{3}ag$$

$$\boxed{\therefore v = \sqrt{\frac{14}{3}ag}}$$

Year 2 Oblique Impacts - successive collisions, oblique impacts with fixed surface

7 A small ball is projected with speed 14 m s^{-1} from a point O on the ground. The ball is projected at an angle α to the ground, where $\tan\alpha = \frac{3}{4}$. The ball bounces on the ground for the first time at the point A_1. The coefficient of restitution between the ball and the ground is $\frac{1}{2}$. The ball is modelled as a particle moving freely under gravity from O to A_1 and between bounces. The ground is modelled as a smooth horizontal plane.

(*a*) Find the size of the angle between the direction of motion of the ball and the ground immediately after the ball bounces on the ground at A_1 **(4)**

(*b*) Explain how, in your calculation, you have used the fact that the ball is moving freely under gravity from O to A_1 **(1)**

The ball bounces on the ground for the second time at the point A_2

(*c*) Find the total time taken by the ball to travel from O to A_2 **(4)**

The ball bounces on the ground for the nth time at the point A_n

Immediately after the ball bounces at A_n, the angle between the direction of motion of the ball and the ground is ϕ.

(*d*) Find, in terms of n only, an expression for $\tan\phi$. **(3)**

(*e*) Describe, according to the model, the subsequent motion of the ball after it has bounced on the ground at A_2 **(1)**

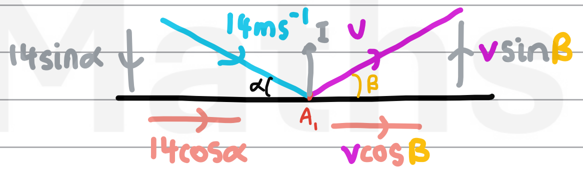
Given instead that the coefficient of restitution between the ball and the ground is 0

(*f*) describe fully the motion of the ball from the instant when it is projected from O. **(2)**

(Total for Question 7 is 15 marks)

(a) first, let's illustrate our first elastic collision with the fixed surface diagrammatically:

COLLISION 1



↳ see that, to get the angle β, we need to find the parallel and perpendicular components of v

... parallel: no impulse ∴ no change:

$v\cos\beta = 14\cos\alpha$

... perpendicular:

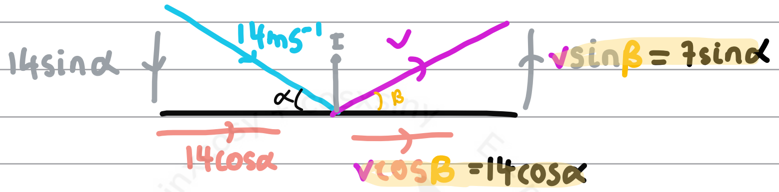
impulse acts perp. to the fixed surface ∴ NEL rearranged applies

$v\sin\beta = e14\sin\alpha$

$$= \frac{1}{2}14\sin\alpha$$

$$= 7\sin\alpha$$

populating this onto our diagram:



$$\therefore \text{see that } \tan\beta = \frac{7\sin\alpha}{14\cos\alpha}$$

using fact that $\frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

$$\Rightarrow \tan\beta = \frac{1}{2}\tan\alpha$$

where $\tan\alpha = 3/4$ (given in question)

$\therefore$ subbing into above

$$\Rightarrow \tan\beta = \frac{1}{2}(3/4)$$

$$= 3/8$$

$$\therefore \beta = \tan^{-1}(3/8)$$

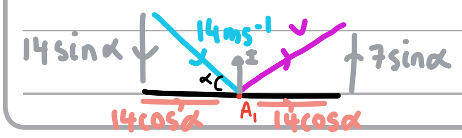
$$= 20.556...^\circ$$

$$\boxed{= 20.6^\circ (3 s.f)}$$

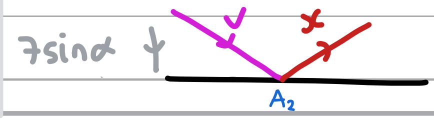
(b) the fact that the ball moves freely under gravity means there is no air resistance $\therefore$ motion is symmetrical: the vertical component down at A_1 is equal to the vertical component up at O

(c) let's split A's journey from O to A_2 into two parts and evaluate SUVAT at each

① from O to A_1



② from A_1 to A_2



considering vertical motion:

$(\uparrow +)$

$u = -14\sin\alpha$

$v = 14\sin\alpha$

$a = -g$

$t = t_1$

formula : $v = u + at$

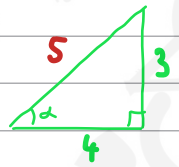
Subbing into above

want to solve for t_1!

$14\sin\alpha = -14\sin\alpha + (-g)(t_1)$

$$\Rightarrow t_1 = \frac{2(14\sin\alpha)}{g}$$

and we can deduce the value of $\sin\alpha$ from $\tan\alpha = 3/4$ - using 3-4-5 Pythag. triple



$\Rightarrow \sin\alpha = 3/5$

$\cos\alpha = 4/5$

$$\therefore t_1 = \frac{2(14(3/5))}{g}$$

$$= \frac{12}{7}$$

...considering vertical motion:

$(\uparrow +)$

$u = -7\sin\alpha$

$v = 7\sin\alpha$

$a = -g$

$t_2 = t_2$

formula: $v = u + at$

subbing into above:

want to solve for t_2!

$7\sin\alpha = -7\sin\alpha + (-g)(t_2)$

$$\Rightarrow t_2 = \frac{2(7\sin\alpha)}{g}$$

$$= \frac{2(7 \times 3/5)}{g}$$

$$= \frac{6}{7}$$

$$\therefore t_1 + t_2 = \frac{12}{7} + \frac{6}{7} = \boxed{\frac{18}{7} \text{ or } 2.57\text{s}}$$

(d) now we're being asked to consider a general n-bounces case -

... parallel component: no Impulse $\Rightarrow$ no change

$14\cos\alpha$

... perp. component: NEL rearranged applies

so $14\sin\alpha \times \left(\frac{1}{2}\right)^n$

$$\therefore \tan\theta = \frac{14\sin\alpha\left(\frac{1}{2}\right)^n}{14\cos\alpha}$$

$$= \tan\alpha\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \tan\theta = 3/4\left(\frac{1}{2}\right)^n = \frac{\frac{3}{4}}{2^n} = \frac{3}{\overset{2^2}{4} \times 2^n} = \boxed{\frac{3}{2^{n+2}}}$$

(e) the ball continues to bounce with the same angle to the ground

(f) after hitting the ground at A_1, the ball will move along the ground (with no rebound) at $14\cos\alpha = 14\left(4/5\right)$

$\boxed{= 11.2\,\text{ms}^{-1}}$